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## A Systematic Study of Epidermal Ridge Minutiae

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**ABSTRACT:** To permit the testing of a number of basic hypotheses regarding the relationship between neighboring minutiae, epidermal ridge minutiae on the distal portions of 412 thumbprints have been surveyed. For each thumbprint a centrally located *focal minutia* was chosen; neighboring minutiae were then sampled. Minutiae were considered to be neighbors if there were no other minutiae in a rectangular region defined by the two minutiae and the ridge system. For each minutia the following data were recorded: minutia type, orientation, ridge count from the focal minutia, and the intervening distance along the ridge flow.

The number of neighbor minutiae was found to be normally distributed. Minutia orientation frequencies on the right and left hands were found to be unequal and to show a mirror image relationship. The incidence of particular minutia orientations and types were found to vary with ridge count from the focal minutia. Linear descriptions of these relationships were found to be adequate after adjustments were made for geometrical restrictions present for ridge counts of one, zero, and negative one. Predicted frequencies of minutia types and orientations for each ridge count were found to be in acceptable agreement with the observed frequencies.

**KEYWORDS:** forensic science, fingerprints, individuality, epidermal ridges, pattern, minutiae

A systematic study of epidermal ridge minutiae benefits forensic science in two major ways: by providing an objective assessment of current fingerprint comparison practices and by extending our ability to evaluate the individuality present in partial fingerprints.

With such a systematic study in mind, the authors have previously proposed a method for description of minutia configurations [1]. The method uses the features of minutia configurations that are directly relevant to the fingerprint comparison process. The present work employs this descriptive method to investigate types and distribution of epidermal ridge minutiae on the distal portion of 412 male thumbprints.

### Descriptive Method

Details of the descriptive method and the rationale for the choice of variables have been fully discussed elsewhere [1]. A brief summary is presented here to introduce terminology and major concepts.

The minutia pair is the fundamental unit of fingerprint variation. The pair consists of not

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only the two minutia events, but also their spatial relationship within the epidermal ridge pattern. Minutiae are thus characterized by type, orientation, and position relative to neighboring minutiae.

The *type* is the form of the minutia. Three fundamental types are used: the fork (or bifurcation); the ending ridge; and the dot. Other "compound" forms of minutiae occur when the fundamental types are in close proximity to one another. The code letters B, E, and D, are assigned to the minutia types of fork (bifurcation), ending ridge, and dot.

*Orientation* refers to the direction along the ridge flow. Minutiae may have one of two orientations relative to the ridge flow. These are assigned by aligning the ridges horizontally. As one follows the ridges from left to right, minutiae that produce new ridges are denoted as positive (P) and minutiae that consume ridges are denoted as negative (N). Dots have no effect on the number of ridges and have no directional sense. They are assigned a null orientation, designated by the letter O.

*Relative position* is described using two variables: the number of ridges separating the second minutia from the first, hereafter referred to as the *ridge count*, and the *distance* between the first minutia and the second along the ridge flow. Only the relative positions of *neighboring* minutiae are described by this system. Minutiae are neighbors when no other minutiae appear in the region between the two.

*Ridge counts* are made in the direction perpendicular to ridge flow from the first minutia to the ridge on which the second minutia appears. Counts to forks are made to the nearest branch, and counts to ending ridges are made as if the ending ridge extended past the counting line. Ridge counts are coded by their direction and magnitude. An upward count of X ridges is assigned a code PX and a downward count of Y ridges is assigned a code NY. Ridge counts of zero occur in conjunction with directions of positive, negative, or zero. A ridge count coded 00 indicates that the two minutiae occupy the same vertical station in relationship to the horizontal ridge flow. Ridge counts of P0 and N0 indicate that the two minutiae are connected to one another by the ridge system, but that their vertical stations are not equivalent. Examples of ridge counts are given in Fig. 1.

Measurement of *distance* is made along ridges using a ridge interval unit. The distance is measured on the ridge of the second minutia, from the place where the perpendicular ridge counting line meets the ridge to the position of the second minutia.

*Minutia pairs* are concisely described by the general formula:

$$(O1) (T1) (O2) (T2) (RC) (I)$$

where

- O1 and O2 = orientations of the first and second minutiae,
- T1 and T2 = corresponding minutia types,
- RC = the ridge count, and
- I = intervening distance.

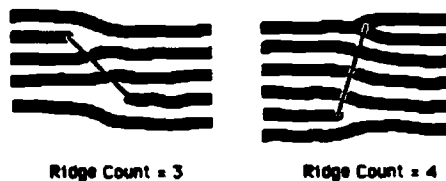


FIG. 1—Simple ridge counts between minutiae. Note that it is actually the intervals between ridges that are counted.

## Methods

### *Source of Sample Material*

Data was collected from inked fingerprints on file at the Contra Costa County Criminalistics Laboratory in Martinez, California. Right and left thumbprints of adult males were examined from sequential fingerprint cards in a file sequenced by date of printing. The cards were screened for printing clarity of the plain thumbprints at the bottom of the standard fingerprint card. The rationale for selecting the plain prints is that they show less distortion than the rolled prints and better approximate the form of an ideal unknown print. Sampling was restricted to male thumbprints to avoid substantial discrepancies in finger size.

In all, 206 pairs of thumbprints were sampled, with a total of 2645 neighbor minutiae. Ages of the subjects ranged from 18 to 56 years (mean = 26.5, standard deviation = 6.5). There were 147 whites, 56 blacks, and 3 Asians in this sample population.

## Data Collection

### *Choice of Area*

The upper portion of the inked prints was chosen as the area for data collection. Fingerprints from this area are commonly encountered; furthermore, all digits and all fingerprint pattern types show a similar ridge flow in this region. The ridges conform to the finger outline, flowing in an arch from one side of the finger to the other. The central portion of loop and whorl patterns is proximal to this region.

### *Selection of Focal Minutiae*

For each thumbprint a centrally located *focal minutia* was chosen using the following procedure (see Figs. 2 and 3). A reticle with a 5-mm circle of radius was placed over the print. This reticle was placed so that the outline of the circle coincided with the upper portion of the ridge most closely corresponding to this degree of curvature. This ridge was designated the *5-mm ridge*. The center of the circle was marked with a fine pinprick on the fingerprint card and was designated the *origin*. (Typically, the origin was in the patterned area of the fingerprint.) The ridges show their highest degree of curvature as they round the fingertip, flattening out as they approach either side. The *point of maximum distal extent* on the 5-mm ridge was thus easily noted and marked with a second pinprick.

A second reticle, marked with angular measure, was placed with the vertex at the origin. The search for a focal minutia began on the 5-mm ridge. The minutia closest to the point of maximum distal extent was selected if one was within 45°. If a minutia was not found within 45° of this point, then the search continued on successively more distal ridges within the total 90° arc until one was found.

### *Sampling of Neighbor Minutiae*

Once a focal minutia was chosen, neighbor minutiae were sampled from within 45° of the focal minutia and within ridge counts of N6 to P9 (see Fig. 4). Below the N6 ridge count, one approaches the pattern area and the ridges cannot be reasonably approximated as circular about the origin. Ridge counts above P9 could not be sampled on all prints as a result of the variable extent of printing. For each neighbor minutia the type, orientation, ridge count, and *angular distance* from the focal minutia was recorded.



FIG. 2—Placement of the reticle on print to be sampled. The 5-mm diameter circular reticle is placed so that the outline of the circle coincides with the upper portion of the ridge most closely corresponding to this degree of curvature.



FIG. 3—Selection of the focal minutia. The focal minutia is selected from within a 90° sector centered where the ridges are at maximum curvature. If no minutia is found on the 5-mm ridge, successively more distal ridges are searched until a focal minutia is found.



FIG. 4—Selection of neighbor minutiae. Neighbor minutiae are selected with ridge counts ranging from N6 to P9 within a 90° sector centered on the focal minutia.

### *Finger Data*

Additional data collected for each finger were the hand (right or left); the general pattern type (arch, radial loop, ulnar loop, or whorl); the type and orientation of the focal minutia; the total number of neighbor minutiae; a measurement of the ridge density; and a measurement of minutia density.

Ridge density was measured along the direction defined by the origin and the focal minutia. The distance between the ridge located five ridges below the focal minutia and that found five ridges above the focal minutia was measured (see Fig. 5).

Minutia density was measured by sampling all minutiae in a circular region with 3-mm radius, centered one ridge directly above the position of maximum distal extent. (This position was chosen because the average focal minutia position was not on the 5-mm ridge, but at 0.91 ridge intervals above it.)

### *Statistical Methods*

Chi square tests were used to test hypotheses of independence, homogeneity, and goodness-of-fit. The Welch-Aspin test and its K-sample analog were used to test for equality of means. Post hoc tests were conducted by Scheffe's method. Least squares regressions were tested for nonzero slope using the *F*-test and a generalized *F*-ratio was used to compare competing nested models. Each of these methods is fully described by Marascuilo [2].

## **Results and Discussion**

### *Frequencies of Pattern Types*

Table 1 shows the frequencies of pattern types in the sample, classified by hand and by racial group. The six prints from Asians have been included with the whites. The expected



FIG. 5—Measurement of ridge density. Ridge density is measured along the direction defined by the origin and the focal minutia. The distance between the ridge five ridges below the focal minutia and the ridge five ridges above the focal minutia is measured.

TABLE 1—Frequencies of fingerprint patterns in the sample (expected values in parentheses).

| Pattern Type            | Whites     |            | Blacks    |            | Total |
|-------------------------|------------|------------|-----------|------------|-------|
|                         | Left Hand  | Right Hand | Left Hand | Right Hand |       |
| Arches/<br>radial loops | 5 (5.1)    | 4 (2.2)    | 4 (6.1)   | 2 (3.6)    | 15    |
| Whorls                  | 50 (44.8)  | 67 (64.6)  | 26 (16.4) | 31 (24.0)  | 174   |
| Ulnar loops             | 95 (100.2) | 79 (83.2)  | 26 (33.4) | 23 (28.3)  | 223   |
| Total                   | 150        | 150        | 56        | 56         | 406   |

frequencies are taken from Plato et al. [3] and from Steinberg et al. [4]. These researchers surveyed fingerprint pattern types by digit and by hand for both American Caucasians and American Negroes. The fingerprint pattern frequencies observed in the present sample are in good statistical agreement with the values expected from these two studies as calculated in Table 1 ( $p = 0.19$ ). We conclude that the sampling is representative of pattern types, given the racial composition of the sample.

*Ridge Density*

The mean ridge density for the 412 thumbprints was 0.463 mm/ridge. Traditionally, ridge density has been expressed in ridges/cm. This value is 21.6 ridges/cm with a variance of 7.95. Table 2 compares the present results with those of Kingston [5] and those of Cummins et al. [6]. Kingston's data was from 100 ulnar loops, 56 from the right hand and 44 from the left, with sex and finger unspecified. The data of Cummins et al. was from the right and left thumbs of 200 males. Both of the values have been adjusted to correct for differences in the measuring technique. Cummins et al. counted the number of ridges crossing a 1-cm line

TABLE 2—*Ridge density as measured in three studies.*

|                    | Sample Size | Mean Number<br>of Ridges<br>per cm | Variance |
|--------------------|-------------|------------------------------------|----------|
| Cummins et al. [6] | 400         | 21.45                              | 4.20     |
| Kingston [5]       | 100         | 21.21                              | 6.28     |
| Present study      | 412         | 21.59                              | 7.95     |

segment. This procedure results in an average loss of 0.5 ridges per measurement when compared with the present study. Kingston measured the number of ridges crossing a 5-mm line segment. His procedure results in an average loss of 0.5 ridges for the 5-mm line segment, which becomes a 1.0 ridge loss for a 1-cm line segment. Kingston's variance was adjusted by a factor of 4 because his mean value was doubled. After these adjustments are made there is no statistical difference between the three mean values in Table 2 ( $p = 0.40$ ).

### *Minutia Density*

For each of the 412 fingerprints the number of minutiae occurring in the circular sampling region of 3-mm radius was recorded. The mean number of minutiae was 6.29, with a standard deviation of 2.61 (sample variance = 6.80). The area of the circular sampling region was 28.3 mm<sup>2</sup>, giving a density estimate of 0.223 minutiae per mm<sup>2</sup> with a 95% confidence interval of 0.215 to 0.230 for the mean.

Table 3 compares these results with those of other investigators [7-12]. With the exception of Trauring's figure, which he deliberately chose as a maximum, the density estimates are close to one another, ranging from 0.181 to 0.295. Given the sample sizes, however, the mean values are not statistically close. For the two other studies estimating variability [10,12] the hypothesis of equal means is soundly rejected ( $p \ll 0.001$ ). Post hoc multiple contrasts demonstrate that each of the three means is significantly different from the other two.

There are two probable causes for the observed range in density values: differences in the fingers and pattern types sampled and differences in the area of the fingerprint sampled. Kingston's sampling [10] was for ulnar loops and the area sampled was a square region, 1 cm on a side, centered at the symmetrical core of the loop pattern. The particular fingers sampled were not specified. In contrast, Dankmeijer et al. [12] sampled all ten fingers and all pattern types. They sampled a square region 2 cm on a side, which is essentially the entire fingerprint. Osterburg et al. [11] also sampled entire fingerprints, but did not specify the pattern types or the fingers sampled. Their sample size was small, and no estimate was made of variability. In the present study all pattern types were included, but only thumbs were

TABLE 3—*Minutia densities observed in seven studies.*

|                        | Sample Size | Minutiae per<br>mm <sup>2</sup> | Standard<br>Deviation |
|------------------------|-------------|---------------------------------|-----------------------|
| Galton [7]             | unspecified | 0.181                           | ...                   |
| Amy [8]                | unspecified | 0.200                           | ...                   |
| Trauring [9]           | unspecified | 0.494 (max)                     | ...                   |
| Kingston [10]          | 100         | 0.246                           | 0.0084                |
| Osterburg et al. [11]  | 39          | 0.295                           | ...                   |
| Dankmeijer et al. [12] | 1000        | 0.190                           | 0.0069                |
| Present study          | 412         | 0.223                           | 0.0045                |

sampled. The sampling region was a small circular region on the distal portion of the thumbprint.

The importance of these differences is demonstrated in Table 4, which shows Kingston's results for different areas within ulnar loops and the results of Dankmeijer et al. for different fingers and for different pattern types. Given Kingston's finding that the density of minutiae is much higher in the core region of the fingerprint, it is not surprising that his value for minutia density is higher than that of most other investigators. He sampled a relatively small region, centered on the core. Dankmeijer et al., on the other hand, included additional patternless areas outside the core and delta regions, resulting in a lower density.

Kingston's results are of interest and demonstrate considerable variation in minutia density within fingerprints. The work of Dankmeijer et al. is complementary. They tested for differences in mean values among the different fingers, hands, and pattern types. Both pattern type and finger were found to significantly affect minutia density, but no significant differences were found between hands.

Of the density values listed in Tables 3 and 4, two are of particular relevance to the present study: the value of Dankmeijer et al. for thumbs, and Kingston's value for the area above the fingerprint core. The former is significantly lower than the density observed here ( $p < 0.02$ ). The area sampled in this study therefore has a significantly greater minutia density than do entire thumbprints. Kingston's value for the area above the fingerprint core does not contradict this. Although it is virtually the same as the value of Dankmeijer et al. for thumbs, the hypothesis of equal means is not rejected because of Kingston's small sample size ( $p = 0.81$ ). This statistical test has very little meaning, however, because a difference in densities of nearly 0.16 would be needed to reject the null hypothesis at 95% significance.

Variation of density with pattern type is of possible concern in the present study, given the results of Dankmeijer et al. In our sampling, however, identical values were obtained for the 225 loops and for the 174 whorl patterns (0.222). We see no differences in minutia density for these two pattern types in our sample region.

*Number of Neighbor Minutiae Sampled Per Print*

The distribution of the number of neighbor minutiae sampled per print is shown in Fig. 6. A total of 2645 neighbor minutiae were sampled from the 412 thumbprints. The number of

TABLE 4—Minutia densities for specific regions, patterns, and fingers.

|                             | Sample Size | Minutiae per mm <sup>2</sup> |
|-----------------------------|-------------|------------------------------|
| Kingston [10]               |             |                              |
| Area above core of loops    | 23          | 0.203                        |
| Core of loops/delta regions | 23          | 0.687                        |
| Dankmeijer et al. [12]      |             |                              |
| Arch patterns               | 63          | 0.159                        |
| Loop patterns               | 594         | 0.183                        |
| Whorl patterns              | 343         | 0.206                        |
| Thumbs                      | 200         | 0.207                        |
| Index fingers               | 200         | 0.177                        |
| Middle fingers              | 200         | 0.183                        |
| Ring fingers                | 200         | 0.200                        |
| Little fingers              | 200         | 0.182                        |



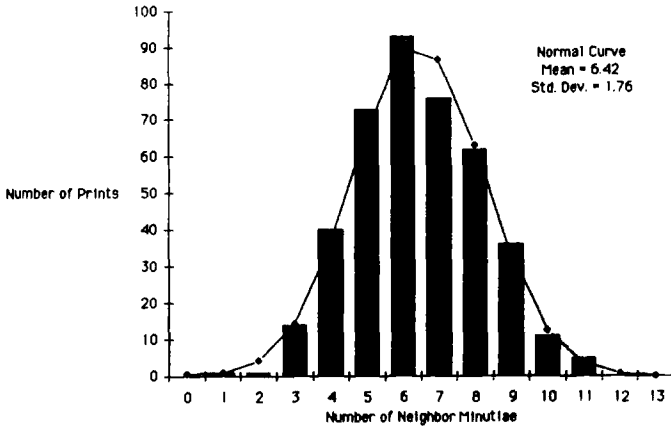


FIG. 6—The distribution of the number of neighbor minutiae per print. The line shows the expected values for a normal distribution with mean 6.42 and standard deviation 1.76.

neighbor minutiae about a focal minutia was found to be normally distributed with mean 6.42 and standard deviation 1.76 ( $p = 0.59$ ).

*Minutia Type Frequencies*

Minutia type frequencies have been measured by forensic scientists [8,11,13-16] and by geneticists [17,18]. Data from these investigators are presented in Tables 5 and 6, together with the frequencies of minutia types found for neighbor minutiae in this study. Table 5 presents relative frequencies of forks and ending ridges; Table 6 presents dot frequencies.

For the five studies with population data [13,14,16-18] the hypothesis of equal proportions of ending ridges is rejected ( $p < 0.001$ ). Okajima's data is clearly divergent from the other four studies, possibly a result of the racial makeup of his sample (entirely Japanese). Even among the other four studies, however, a test of equal proportions fails at high significance ( $p < 0.001$ ). Post hoc tests of all pairwise comparisons result in rejection of the hypothesis of equal proportions for all but the present study and Roxburgh's. The correspondence with Roxburgh's data is particularly interesting in that Roxburgh collected his

TABLE 5—Relative frequencies of forks and ending ridges in eight studies.

| Type                              | Roxburgh <sup>a</sup><br>[14] | Amy<br>[8] | Santa-<br>maria <sup>b</sup><br>[15] | Kingston <sup>b</sup><br>[13] | Okajima <sup>c</sup><br>[17] | Loesch <sup>d</sup><br>[18] | Sclove <sup>b</sup><br>[16] | Present<br>Study |
|-----------------------------------|-------------------------------|------------|--------------------------------------|-------------------------------|------------------------------|-----------------------------|-----------------------------|------------------|
| Ending ridges                     | 0.589                         | 0.60       | 0.653                                | 0.509                         | 0.837                        | 0.618                       | 0.662                       | 0.568            |
| Forks                             | 0.411                         | 0.40       | 0.347                                | 0.491                         | 0.163                        | 0.382                       | 0.338                       | 0.432            |
| Number of<br>minutiae<br>surveyed | 1088                          | ?          | ?                                    | 2428                          | 10 462                       | ?                           | 3053                        | 2645             |

<sup>a</sup>Roxburgh's data were adapted from his Table 2, decomposing his types five through ten into the fundamental minutia types.

<sup>b</sup>Santamaria's, Kingston's and Sclove's data have been adapted by decomposing the compound minutia types into the fundamental types.

<sup>c</sup>Okajima's data are for thumbs only.

<sup>d</sup>Loesch's data are the weighted average for the pattern types listed in her Table 2.

TABLE 6—Relative frequency of dots in four studies.

| Study         | Relative Frequency of Dots |
|---------------|----------------------------|
| Kingston [13] | 0.084                      |
| Okajima [17]  | 0.050                      |
| Sclove [16]   | 0.084                      |
| Present study | 0.053                      |

data only from the portion of the fingerprint directly above the pattern core; this is the region in which the neighbor minutiae were sampled in this study. Finding this correspondence is very reasonable and demonstrates reproducibility in the proportion of ending ridges for this region of the fingerprint. The deviations seen with and among the other studies demonstrate that minutia type frequencies are, in general, highly variable.

The frequencies of dots in three prior studies [13, 16, 17] and the present one are presented in Table 6. The hypothesis of equal proportions is rejected at high significance ( $p \ll 0.001$ ). Using post hoc tests we find that the studies fall into two groups, for which the proportions are accepted as being equal. Okajima's frequency is not statistically different from the present sampling. Sclove's and Kingston's frequencies are not statistically different from each other. The methodological differences cited earlier are a likely source of the variation observed.

*Minutia Types and Fingerprint Patterns*

Okajima [17] made an extensive survey of the proportion of forks to ending ridges and compared this proportion among the different fingers and pattern types. He found that ulnar loops and whorls differed in this ratio for most fingers, with forks being more favored in loops. Interestingly, thumbs of males did not show any differences in this proportion among the pattern types. Okajima did find significant differences in the proportion among the different fingers and between the sexes.

Loesch [18] also surveyed the proportion of forks to ending ridges. She sampled ring fingers and analyzed her data with respect to pattern type. There was no significant difference in this proportion among the pattern types.

The results of these two investigators are in agreement with those in the present study. Table 7 shows the minutia type frequency results by pattern type. The chi square test for independence of minutia type and pattern fails ( $p < 0.001$ ), but of the eleven cells in the test, ten contribute a total of 2.80 to the chi square statistic, whereas the last cell gives 18.52. This high value is attributable to 2 fingerprints among the 412 sampled. Two arch patterns had four dots each, contributing eight of the fourteen dots observed in all the arch patterns. If these two prints are removed, independence is accepted with high significance ( $p = 0.83$ ). Given these results and absent further data regarding the frequencies of dots in arch patterns, it would be reckless to reject the hypothesis of independence between the pattern type and the minutia type frequencies. The data in fact provide considerable support for independence.

*Minutia Orientation Frequencies*

The orientation of minutiae is determined by whether a ridge is produced or consumed as one follows along the ridge flow from left to right. Differences in frequencies between the two minutia orientations have been previously considered only in the studies of Amy [8] and of

TABLE 7—Frequencies of minutia types seen in the different fingerprint patterns (frequencies expected under an assumption of independence are in parentheses).

| Fingerprint Pattern Type                        | Ending Ridges | Minutia Type |           |
|---|---------------|--------------|-----------|
|   |               | Forks        | Dots      |
| Arch  | 41 (43.8)     | 26 (32.5)    | 14 (4.7)  |
| Radial loop                                     | 6 (5.4)       | 3 (4.0)      | 1 (0.6)   |
| Ulnar loop                                      | 772 (765.0)   | 568 (567.2)  | 74 (81.8) |
| Whorl   | 612 (616.8)   | 464 (457.3)  | 64 (65.9) |
| Sum of squares = 21.32                          |               |              |           |
| Effect of removing two of the 406 fingerprints: |               |              |           |
| Arch  | 35 (35.8)     | 25 (26.6)    | 6 (3.6)   |
| Radial loop                                     | 6 (5.4)       | 3 (4.0)      | 1 (0.6)   |
| Ulnar loop                                      | 772 (766.1)   | 568 (569.9)  | 74 (78.0) |
| Whorl   | 612 (617.7)   | 464 (459.5)  | 64 (62.9) |
| Sum of squares = 2.14                           |               |              |           |

Roxburgh [14]. Other investigators have either assumed equal frequencies for the two orientations without experimental justification [9, 19], or have not considered the issue of minutia orientation [5, 11, 20].

In the present study the 2492 neighbor minutiae (other than dots) showed 1249 with positive orientation and 1243 with negative orientation. This apparent equality of orientation is deceptive. A mirror image relationship exists between the left and right hands—a relationship that extends to many features of the fingerprint patterns [21]. The classification of loops as radial and ulnar is a familiar example. We might well expect a mirror image relationship between the minutia orientations on the right and left hands, and the data in Table 8 demonstrates that this is, in fact, the case. Fingerprints from the right hand show an excess of minutiae that result in the loss of a ridge when one follows the ridge flow from right to left. The opposite is the case for fingerprints from the left hand. The difference is obviously highly significant. If we perform a mirror image transformation on the prints from the left hand, however, the data become as shown in Table 9. After this transformation the proportions of positive and negative orientations on each hand are found to be statistically equivalent ( $p = 0.48$ ).

The combined frequencies for the right hand and the transformed left hand are presented in Table 10, together with Roxburgh's and Amy's data. Roxburgh's data do not specify the hand, and thus it is quite possible that results from the left and right hands are pooled to some degree, accounting for the discrepancy. Amy's data are in good agreement with the present study ( $p = 0.31$ ).

TABLE 8—Frequencies of minutia orientations on each hand.

| Hand       | Minutia Orientation |          |       |
|------------|---------------------|----------|-------|
|            | Positive            | Negative | Total |
| Left hand  | 950                 | 333      | 1283  |
| Right hand | 299                 | 910      | 1209  |
| Total      | 1249                | 1243     | 2492  |

TABLE 9—Frequencies of minutia orientations with the left hand orientations reversed.

| Hand                    | Minutia Orientation |                       | Total |
|-------------------------|---------------------|-----------------------|-------|
|                         | Positive            | Negative <sup>a</sup> |       |
| Left hand<br>(Reversed) | 333                 | 950                   | 1283  |
| Right hand              | 299                 | 910                   | 1209  |
| Total                   | 632                 | 1860                  | 2492  |

<sup>a</sup>Pooled estimate for proportion of negative minutiae = 0.746.

TABLE 10—Minutia orientation frequencies in three studies.

| Study         | Number of Minutiae | Proportion with Negative Orientation |
|---------------|--------------------|--------------------------------------|
| Roxburgh [14] | 1088               | 0.579                                |
| Amy [8]       | 2000               | 0.733                                |
| Present study | 2492               | 0.746                                |

*Independence of Minutia Type and Orientation*

Table 11 presents a 2 by 2 contingency table for testing the independence of minutia type and orientation. Formal application of the chi square test rejects independence ( $p = 0.001$ ). This rejection, however, may well be an artifact of sample size [22], and the correlation coefficient must be considered in evaluation of independence. The correlation coefficient is, in fact, only 0.065, which demonstrates virtual independence of these two parameters.

*Relationship of Minutia Position, Orientation, and Type*

*Minutia Orientation, Type, and Distance*—Table 12 gives the observed incidence of minutia types and orientations in 5° sectors within the sampling region. Sector Number 1 is the first 5° sector from the focal axis, Sector Number 2 is from 5 to 10°, and so forth. The

TABLE 11—Independence of minutia type and orientation (expected frequencies in parentheses).<sup>a</sup>

| Type         | Orientation |           | Total |
|--------------|-------------|-----------|-------|
|              | Negative    | Positive  |       |
| Ending Ridge | 1033 (1068) | 398 (363) | 1431  |
| Fork         | 827 (792)   | 234 (269) | 1061  |
| Total        | 1860        | 632       | 2492  |

Sum of squares = 10.66

<sup>a</sup>Estimated correlation coefficient = 0.065.

TABLE 12—Incidence of minutia types and orientations in 5° sectors.

| Sector Number | Sign of Distance | Orientation and Type (OT) |     |      |      |      | Total |
|---------------|------------------|---------------------------|-----|------|------|------|-------|
|               |                  | NE                        | NB  | PE   | PB   | OD   |       |
| 1             | >0               | 81                        | 76  | 52.5 | 20   | 18   | 247.5 |
|               | <0               | 92                        | 55  | 33.5 | 22   | 16   | 218.5 |
| 2             | >0               | 89                        | 66  | 30   | 16   | 12   | 213   |
|               | <0               | 85                        | 57  | 45   | 22   | 21   | 230   |
| 3             | >0               | 73                        | 65  | 34   | 13   | 8    | 193   |
|               | <0               | 72                        | 65  | 30   | 16   | 12   | 195   |
| 4             | >0               | 56                        | 47  | 19   | 14   | 6    | 142   |
|               | <0               | 58                        | 56  | 18   | 11   | 5    | 148   |
| 5             | >0               | 47                        | 35  | 17   | 17   | 7    | 123   |
|               | <0               | 59                        | 25  | 21   | 14   | 7    | 126   |
| 6             | >0               | 52                        | 41  | 18   | 5    | 5    | 121   |
|               | <0               | 50                        | 40  | 16   | 12   | 9    | 127   |
| 7             | >0               | 39                        | 26  | 6    | (10) | (3)  | 84    |
|               | <0               | 41                        | 47  | 14   | (10) | (5)  | 117   |
| 8             | >0               | 36                        | 38  | 17   | (9)  | (3)  | 103   |
|               | <0               | 38                        | 30  | 6    | (6)  | (3)  | 83    |
| 9             | >0               | 27                        | 25  | 11   | (13) | (2)  | 78    |
|               | <0               | 38                        | 33  | 10   | (4)  | (11) | 96    |
| Total         |                  | 1033                      | 827 | 398  | 234  | 153  | 2645  |

Sum of squares (homogeneity of corresponding sectors) = 43.8.

incidence of minutiae in corresponding sectors on either side of the focal axis (opposite distance signs) are found to be statistically equivalent ( $p = 0.11$ ). Table 13 shows the observed frequencies of minutia type and orientation when the sectors with corresponding positive and negative distances are combined. Expected values, assuming independence of minutia type, orientation, and distance, are given in parentheses. The hypothesis of independence is accepted ( $p = 0.08$ ).

*Minutia Ridge Count, Orientation, and Type*—Table 14 presents observed frequencies of minutia type and orientation for each ridge count. Expected frequencies are given under an assumption of independence. Independence is rejected ( $p \ll 0.001$ ). The value of the corre-

TABLE 13—Incidence of minutia types and orientations in 5° sectors.<sup>a</sup>

| Sector Number | Orientation and Type |           |         |         |         | Total |
|---------------|----------------------|-----------|---------|---------|---------|-------|
|               | NE                   | NB        | PE      | PB      | OD      |       |
| 1             | 173 (182)            | 131 (146) | 86 (70) | 42 (41) | 34 (27) | 466   |
| 2             | 174 (173)            | 123 (139) | 75 (67) | 38 (39) | 33 (26) | 443   |
| 3             | 145 (152)            | 130 (121) | 64 (58) | 29 (34) | 20 (22) | 388   |
| 4             | 114 (113)            | 103 (91)  | 37 (44) | 25 (26) | 11 (17) | 290   |
| 5             | 106 (97)             | 60 (78)   | 38 (37) | 31 (22) | 14 (14) | 249   |
| 6             | 102 (97)             | 81 (78)   | 34 (37) | 17 (22) | 14 (14) | 248   |
| 7             | 80 (78)              | 73 (63)   | 20 (30) | 20 (18) | 8 (12)  | 201   |
| 8             | 74 (73)              | 68 (58)   | 23 (28) | 15 (16) | 6 (11)  | 186   |
| 9             | 65 (68)              | 58 (54)   | 21 (26) | 17 (15) | 13 (10) | 174   |
| Total         | 1033                 | 827       | 398     | 234     | 153     | 2645  |

Sum of squares (independence) = 43.8

<sup>a</sup>Positive and negative distances are combined. Values expected under the hypothesis of independence are in parentheses.

TABLE 14—Observed and expected frequencies of minutia types and orientations by ridge.

| Ridge Number | Orientation and Type |           |           |           |           | Total |
|--------------|----------------------|-----------|-----------|-----------|-----------|-------|
|              | NE                   | NB        | PE        | PB        | OD        |       |
| P9           | 41 (37.1)            | 21 (29.7) | 17 (14.3) | 10 (8.40) | 6 (5.49)  | 95    |
| P8           | 40 (37.5)            | 23 (30.0) | 23 (14.4) | 6 (8.49)  | 4 (5.55)  | 96    |
| P7           | 43 (44.9)            | 32 (36.0) | 23 (17.3) | 8 (10.2)  | 9 (6.65)  | 115   |
| P6           | 62 (59.8)            | 41 (47.8) | 29 (23.0) | 15 (13.5) | 6 (8.85)  | 153   |
| P5           | 69 (60.9)            | 35 (48.8) | 36 (23.5) | 9 (13.8)  | 7 (9.02)  | 156   |
| P4           | 90 (76.6)            | 39 (61.3) | 32 (29.5) | 24 (17.3) | 11 (11.3) | 196   |
| P3           | 87 (82.4)            | 60 (66.0) | 35 (31.8) | 20 (18.7) | 9 (12.2)  | 211   |
| P2           | 110 (100)            | 66 (80.4) | 42 (38.7) | 25 (22.7) | 14 (14.9) | 257   |
| P1           | 79 (89.0)            | 72 (71.3) | 36 (34.3) | 29 (20.2) | 12 (13.2) | 228   |
| 0            | 36 (65.6)            | 71 (52.5) | 26 (25.3) | 23 (14.9) | 12 (9.72) | 168   |
| N1           | 55 (66.4)            | 70 (53.2) | 22 (25.6) | 14 (15.0) | 9 (9.83)  | 170   |
| N2           | 88 (86.7)            | 80 (69.4) | 20 (33.4) | 17 (19.6) | 17 (12.8) | 222   |
| N3           | 86 (81.6)            | 83 (65.4) | 19 (31.4) | 12 (18.5) | 9 (12.1)  | 209   |
| N4           | 73 (70.7)            | 65 (56.6) | 14 (27.2) | 13 (16.0) | 16 (10.5) | 181   |
| N5           | 38 (43.4)            | 39 (34.7) | 19 (16.7) | 5 (9.82)  | 10 (6.42) | 111   |
| N6           | 36 (30.1)            | 30 (24.1) | 5 (11.6)  | 4 (6.81)  | 2 (4.45)  | 77    |
| Totals       | 1033                 | 827       | 398       | 234       | 153       | 2645  |

Sum of squares (independence) = 140.4

lation coefficient is 0.23. Although this is not a strikingly high correlation, it does call for a more critical examination of the relationships involved.

*Relationship of Minutia Orientation to Ridge Count*—Figure 7 shows how the proportion of negative orientations among forks and ending ridges varies with ridge count. The least squares regression line is drawn, and the analysis of variance is presented in Table 15. We reject the hypothesis of zero slope for the regression line ( $p \ll 0.001$ ) and obtain a multiple correlation coefficient of 0.66. Expected frequencies based on this regression line are given in Table 16 together with the observed frequencies. The linear model is accepted ( $p = 0.20$ ).

*Relationship of Dot Frequencies to Ridge Count*—Figure 8 shows the proportion of dots observed for each ridge count. The least squares regression line is drawn and the analysis of variance is presented in Table 17. The slope of the regression line is not significantly different from zero ( $p = 0.48$ ). We therefore proceed under the assumption that the frequency of

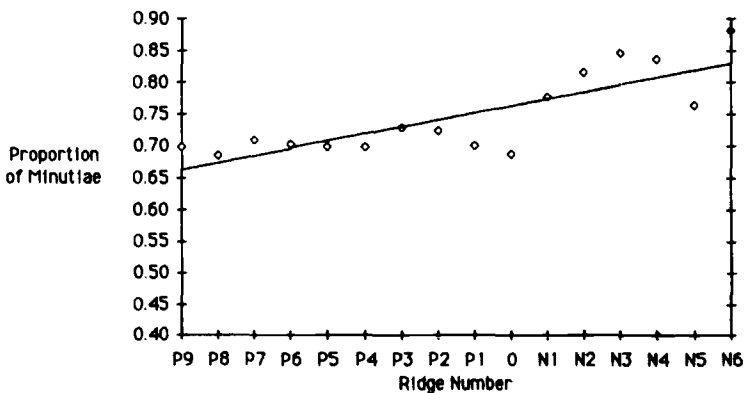


FIG. 7—Proportion of minutiae with negative orientation by ridge count. The least squares regression line is drawn.

TABLE 15—Analysis of variance for orientation regression line in Fig. 7.<sup>a</sup>

|            | Sum of Squares | D.F. | Mean Square | F Ratio |
|------------|----------------|------|-------------|---------|
| Regression | 4.21E-02       | 1    | 4.21E-02    | 27.6    |
| Residual   | 2.14E-02       | 14   | 1.53E-03    |         |
| Total      | 6.35E-02       | 15   |             |         |

<sup>a</sup> $F(1, 14), 0.95 = 4.60$ . Multiple correlation coefficient = 0.66.

TABLE 16—Observed and expected frequencies of minutia orientations on each ridge.

| Ridge Number | Negative  | Orientation Positive | Zero (Dots) | Total |
|--------------|-----------|----------------------|-------------|-------|
| P9           | 62 (63.0) | 27 (32.0)            | 6 (5.1)     | 95    |
| P8           | 63 (61.4) | 29 (29.7)            | 4 (5.1)     | 96    |
| P7           | 75 (74.8) | 31 (34.4)            | 9 (6.1)     | 115   |
| P6           | 103 (101) | 44 (44.1)            | 6 (8.2)     | 153   |
| P5           | 104 (105) | 45 (43.3)            | 7 (8.3)     | 156   |
| P4           | 129 (130) | 56 (56.3)            | 11 (10.4)   | 196   |
| P3           | 147 (146) | 55 (54.2)            | 9 (11.2)    | 211   |
| P2           | 176 (181) | 67 (63.2)            | 14 (13.7)   | 257   |
| P1           | 151 (163) | 65 (53.7)            | 12 (12.2)   | 228   |
| 0            | 107 (122) | 49 (37.8)            | 12 (9.0)    | 168   |
| N1           | 125 (125) | 36 (36.4)            | 9 (9.1)     | 170   |
| N2           | 168 (166) | 37 (45.2)            | 17 (11.8)   | 222   |
| N3           | 169 (158) | 31 (40.4)            | 9 (11.1)    | 209   |
| N4           | 138 (139) | 27 (33.1)            | 16 (9.6)    | 181   |
| N5           | 77 (86.3) | 24 (19.1)            | 10 (5.9)    | 111   |
| N6           | 66 (60.7) | 9 (12.4)             | 2 (4.1)     | 77    |
| Totals       | 1860      | 632                  | 153         | 2645  |

Sum of squares (independence) = 34.0.

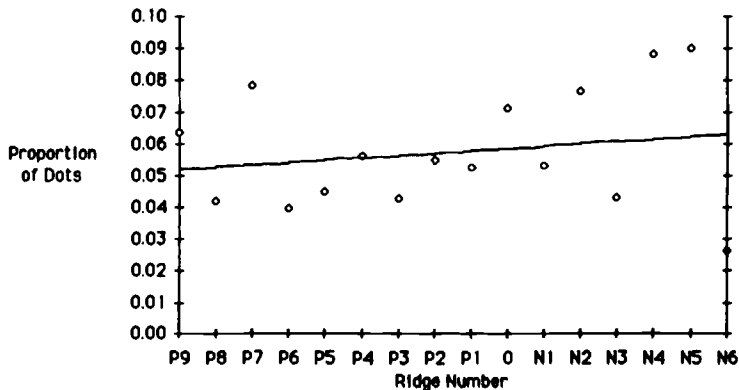


FIG. 8—Proportion of dots by ridge count. The least squares regression line is drawn. The slope of this line is not significantly different from zero.

TABLE 17—Analysis of variance for dot frequency regression line in Fig. 8.<sup>a</sup>

|            | Sum of Squares | D.F. | Mean Square | F Ratio |
|------------|----------------|------|-------------|---------|
| Regression | 1.90E-04       | 1    | 1.90E-04    | 0.52    |
| Residual   | 5.07E-03       | 14   | 3.62E-04    |         |
| Total      | 5.26E-03       | 15   |             |         |

<sup>a</sup> $F(1, 14), 0.95 = 4.60$ . Multiple correlation coefficient = 0.04.

dots is independent of ridge count, and examine the relative frequencies of forks and ending ridges in more detail.

*Relationship of Ending Ridges and Forks to Ridge Count*—Figure 9 shows how the proportion of ending ridges varies with ridge count (dots omitted). The least squares regression line is drawn (Line 1) and the analysis of variance is presented in Table 18. We reject the hypothesis of zero slope for the regression line ( $p < 0.01$ ) and obtain a multiple correlation coefficient of 0.46.

Inspection of Fig. 9 reveals a rather pronounced deviation in the proportion of ending ridges in the vicinity of ridge count zero. Forks are much more frequently encountered with ridge count zero than they are for any other ridge count. There is a good explanation for this—not all combinations of minutia types and orientations are possible with ridge counts of one and zero. Furthermore, with a ridge count of zero, some types of minutiae actually have two possible counts: “positive zero” and “negative zero.” This phenomenon was discussed by Amy [8] and resulted in the correction factor G in his fingerprint model [23]. Table 19 lists these special cases for minutia pairs with ridge counts of 0 and P1. (The relationships for ridge count N1 are the same as for P1.) Table 20 summarizes the effect on the number of ways a given combination of minutia types and orientations may occur.

Adjusted frequencies for minutia type and orientation with ridge counts of P1, 0, and N1 were computed by calculating frequencies conditional on the type and orientation of the

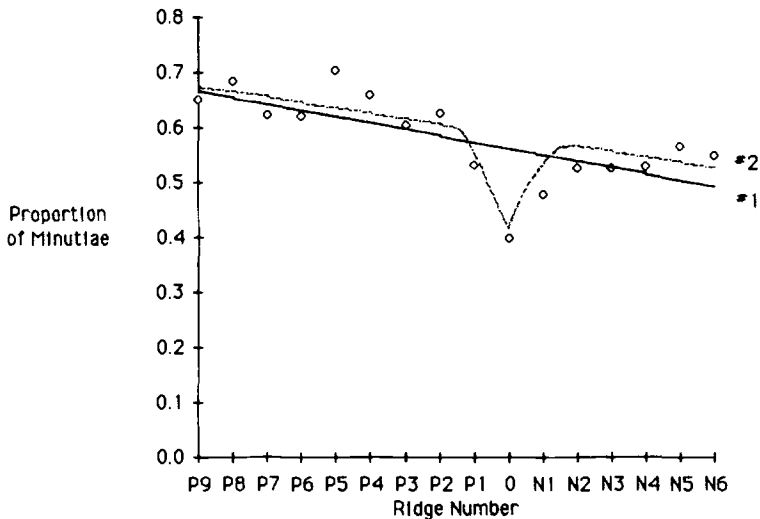


FIG. 9—Proportion of ending ridges by ridge number (dots omitted). Line 1 is the least squares regression line for all ridge counts. Line 2 is that obtained from omission of ridge counts P1, 0, and N1 (see text).



TABLE 18—Analysis of variance for ending ridge frequency regression line #1 in Fig. 9.<sup>a</sup>

|            | Sum of Squares | D.F. | Mean Square | F Ratio |
|------------|----------------|------|-------------|---------|
| Regression | 4.59E-02       | 1    | 4.59E-02    | 11.90   |
| Residual   | 5.40E-02       | 14   | 3.86E-03    |         |
| Total      | 9.99E-02       | 15   |             |         |

<sup>a</sup> $F(1, 14), 0.95 = 4.60$ . Multiple correlation coefficient = 0.46.

TABLE 19—Special cases for minutia pairs on ridges zero and one.

| Focal Minutia  | Neighbor Minutia | Sign of Distance |
|--|------------------|------------------|
| Ridge Count P1 or N1 is NOT ALLOWED under these circumstances: |                  |                  |
| PE   | NE               | negative         |
| PE   | OD               | negative         |
| NE   | PE               | positive         |
| NE   | OD               | positive         |
| OD   | PE               | positive         |
| OD   | NE               | negative         |
| OD   | OD               | positive         |
| OD   | OD               | negative         |
| Ridge Count Zero is NOT ALLOWED under these circumstances:     |                  |                  |
| PB   | NE               | negative         |
| PB   | OD               | negative         |
| PE   | PE               | positive         |
| PE   | PE               | negative         |
| PE   | NB               | negative         |
| PE   | OD               | positive         |
| NB   | PE               | positive         |
| NB   | OD               | positive         |
| NE   | PB               | positive         |
| NE   | NE               | positive         |
| NE   | NE               | negative         |
| NE   | OD               | negative         |
| OD   | PB               | positive         |
| OD   | PE               | negative         |
| OD   | NB               | negative         |
| OD   | NE               | positive         |
| Ridge Count Zero has TWO OPTIONS under these circumstances:    |                  |                  |
| PB   | PB               | positive         |
| PB   | PB               | negative         |
| PB   | NE               | positive         |
| PE   | NB               | positive         |
| NB   | PE               | negative         |
| NB   | NB               | positive         |
| NB   | NB               | negative         |
| NE   | PB               | negative         |
| Ridge Count Zero has THREE OPTIONS under these circumstances:  |                  |                  |
| NB   | PB               | negative         |
| PB   | NB               | positive         |

TABLE 20—Number of possible type and orientation combinations for minutia pairs on Ridges 0, P1, and N1.<sup>a</sup>

| Focal Minutia         | Neighbor Minutia |    |    |    |    |
|-----------------------|------------------|----|----|----|----|
|                       | NE               | NB | PE | PB | OD |
| Ridge Count Zero:     |                  |    |    |    |    |
| NE                    | 2                | 4  | 2  | 4  | 1  |
| NB                    | 2                | 2  | 0  | 2  | 1  |
| PE                    | 2                | 4  | 2  | 4  | 1  |
| PB                    | 0                | 2  | 2  | 2  | 1  |
| OD                    | 1                | 1  | 1  | 1  | 2  |
| Ridge Count P1 or N1: |                  |    |    |    |    |
| NE                    | 2                | 2  | 2  | 2  | 2  |
| NB                    | 1                | 2  | 2  | 2  | 1  |
| PE                    | 2                | 2  | 2  | 2  | 2  |
| PB                    | 2                | 2  | 1  | 2  | 1  |
| OD                    | 1                | 2  | 1  | 2  | 0  |

<sup>a</sup>Under normal circumstances there would be two possibilities for any particular combination of types and orientations: one with the distance >0, and one with the distance <0.

focal minutia and then weighting these frequencies according to the predicted frequency of the focal minutia type and orientation.

Conditional frequencies were calculated by weighting the predicted (regression) frequencies by the number of different ways that the minutiae may occur (from Table 20) and normalizing. Weighting each of these conditional frequencies by the estimated frequency of the focal minutia type and orientation resulted in the overall adjusted frequencies for the particular ridge count.

The adjusted frequencies for ridge counts of P1, 0, and N1 are given in Table 21. Values in parentheses are those predicted by the regression line. With ridge count zero the predicted relative frequency of ending ridges decreases by nearly 25%. For ridge counts of P1 and N1 this frequency decreases by 7 to 8%. Orientation frequencies are less affected: for a ridge count of zero the proportion of negative orientations decreases by only 3.5%, and the changes for ridge counts of P1 and N1 are less than 2%.

Using the adjusted frequencies for the occurrence of ending ridges with ridge counts of P1, 0, and N1, we may test whether the linear model for minutia type frequencies is significantly improved. Linear regression on the remaining data results in regression Line 2 shown in Fig. 9. The analysis of variance for regression is shown in Table 22. The hypothesis of zero slope for the regression line is again rejected ( $p < 0.001$ ), but the multiple correlation coefficient

TABLE 21—Adjusted frequencies for ridge counts of P1, 0, and N1.<sup>a</sup>

| Ridge Number | Relative Frequency $E/E + B$ | Relative Frequency $N/N + P$ |
|--------------|------------------------------|------------------------------|
| P1           | 0.552 (0.596)                | 0.766 (0.752)                |
| 0            | 0.417 (0.586)                | 0.736 (0.763)                |
| N1           | 0.531 (0.576)                | 0.788 (0.774)                |

<sup>a</sup>Values in parentheses are those predicted by regression line 1 in Fig. 9.

TABLE 22—Analysis of variance for ending ridge frequency regression line #2 in Fig. 9.<sup>a</sup>

|            | Sum of Squares | D.F. | Mean Square | F Ratio |
|------------|----------------|------|-------------|---------|
| Regression | 3.28E-02       | 1    | 3.28E-02    | 27.33   |
| Residual   | 1.32E-02       | 11   | 1.20E-03    |         |
| Total      | 4.60E-02       | 12   |             |         |

<sup>a</sup> $F(1, 11), 0.95 = 4.84$ . Multiple correlation coefficient = 0.71.

has increased to 0.71. For the three ridge counts requiring special treatment the sum of squares is 0.0094. There are no additional degrees of freedom (three cells, less three estimated parameters). The two hypotheses are nested, with the full linear regression being the more restricted hypothesis. Using a generalized  $F$ -ratio, special treatment of the three ridge counts is found to significantly improve the model ( $p < 0.01$ ).

*Regression Model for Minutia Type and Ridge Count*—Table 23 presents the observed frequencies of minutia type by ridge count along with the frequencies predicted by the modified linear regression model presented above. The model is accepted ( $p = 0.55$ ).

*Regression Model for Minutia Type, Orientation, and Ridge Count*—Combining the modified linear regression models for type and orientation results in the expected values given in Table 24. These expected values may be compared with those following only from an assumption of independence (Table 14). A generalized  $F$ -ratio demonstrates that the model has been significantly improved ( $p \ll 0.001$ ). The chi square goodness of fit test also accepts the modified regression model ( $p = 0.054$ ). This value of  $p$  is not very impressive and may indicate that there is a more appropriate model for the relationships. The modified regression model, however, does reasonably describe the relationships for these data.

TABLE 23—Observed and expected frequencies of minutia types on each ridge with adjustments for Ridges P1, 0, and N1.

| Ridge Number | Minutia Type |            |           | Total |
|--------------|--------------|------------|-----------|-------|
|              | Ending Ridge | Fork       | Dot       |       |
| P9           | 58 (60.5)    | 31 (29.0)  | 6 (5.5)   | 95    |
| P8           | 63 (60.2)    | 29 (30.2)  | 4 (5.6)   | 96    |
| P7           | 66 (71.1)    | 40 (37.3)  | 9 (6.7)   | 115   |
| P6           | 91 (93.2)    | 56 (51.0)  | 6 (8.9)   | 153   |
| P5           | 105 (93.5)   | 44 (53.5)  | 7 (9.0)   | 156   |
| P4           | 122 (116)    | 63 (69.0)  | 11 (11.3) | 196   |
| P3           | 122 (122)    | 80 (76.3)  | 9 (12.2)  | 211   |
| P2           | 152 (147)    | 91 (95.3)  | 14 (14.9) | 257   |
| P1           | 115 (117)    | 101 (98.1) | 12 (13.2) | 228   |
| 0            | 62 (57.2)    | 94 (101)   | 12 (9.7)  | 168   |
| N1           | 77 (87.0)    | 84 (73.2)  | 9 (9.8)   | 170   |
| N2           | 108 (118)    | 97 (90.7)  | 17 (12.8) | 222   |
| N3           | 105 (110)    | 95 (87.3)  | 9 (12.1)  | 209   |
| N4           | 87 (93.2)    | 78 (77.3)  | 16 (10.5) | 181   |
| N5           | 57 (56.1)    | 44 (48.5)  | 10 (6.4)  | 111   |
| N6           | 41 (38.2)    | 34 (34.4)  | 2 (4.4)   | 77    |
| Totals       | 1431         | 1061       | 153       | 2645  |

Sum of squares (independence) = 25.4

TABLE 24—*Observed and expected frequencies of minutia types and orientations by ridge count (modified regression model).*

| Ridge Number | Orientation and Type |           |           |           |           | Total |
|--------------|----------------------|-----------|-----------|-----------|-----------|-------|
|              | NE                   | NB        | PE        | PB        | OD        |       |
| P9           | 41 (40.1)            | 21 (19.2) | 17 (20.4) | 10 (9.77) | 6 (5.50)  | 95    |
| P8           | 40 (40.6)            | 23 (20.4) | 23 (19.6) | 6 (9.85)  | 4 (5.55)  | 96    |
| P7           | 43 (48.7)            | 32 (25.5) | 23 (22.4) | 8 (11.7)  | 9 (6.65)  | 115   |
| P6           | 62 (64.8)            | 41 (35.5) | 29 (28.3) | 15 (15.5) | 6 (8.85)  | 153   |
| P5           | 69 (66.1)            | 35 (37.8) | 36 (27.4) | 9 (15.6)  | 7 (9.02)  | 156   |
| P4           | 90 (83.1)            | 39 (49.6) | 32 (32.6) | 24 (19.4) | 11 (11.3) | 196   |
| P3           | 87 (89.4)            | 60 (55.7) | 35 (33.1) | 20 (20.6) | 9 (12.2)  | 211   |
| P2           | 110 (109)            | 66 (70.6) | 42 (38.0) | 25 (24.7) | 14 (14.9) | 257   |
| P1           | 79 (85.6)            | 72 (79.1) | 36 (26.1) | 29 (24.1) | 12 (13.2) | 228   |
| 0            | 36 (45.8)            | 71 (70.7) | 26 (16.4) | 23 (25.3) | 12 (9.72) | 168   |
| N1           | 55 (63.2)            | 70 (63.0) | 22 (17.0) | 14 (17.0) | 9 (9.83)  | 170   |
| N2           | 88 (93.0)            | 80 (71.2) | 20 (25.4) | 17 (19.5) | 17 (12.8) | 222   |
| N3           | 86 (87.3)            | 83 (69.6) | 19 (22.3) | 12 (17.8) | 9 (12.1)  | 209   |
| N4           | 73 (75.3)            | 65 (62.5) | 14 (17.9) | 13 (14.9) | 16 (10.5) | 181   |
| N5           | 38 (45.9)            | 39 (39.7) | 19 (10.2) | 5 (8.79)  | 10 (6.42) | 111   |
| N6           | 36 (31.7)            | 30 (28.5) | 5 (6.50)  | 4 (5.84)  | 2 (4.45)  | 77    |
| Totals       | 1033                 | 827       | 398       | 234       | 153       | 2645  |

**Summary**

The findings of this work may be summarized as follows. Strictly interpreted, all findings are limited to epidermal ridges on the distal tip of the male thumb.

1. Minutiae may be described using the methods previously proposed by the authors [1].
2. The ridge densities found were in excellent agreement with Kingston [5] and with Cummins et al. [6].
3. The density of minutiae observed was found to be different from that reported by Kingston [10], by Osterburg et al. [11], and by Dankmeijer et al. [12]. Differences in the size and location of the sampling regions are the probable cause of the discrepancy. The present results support the generalization made by these prior investigators that minutia density varies significantly within epidermal ridge patterns.
4. No variation in minutia density with pattern type was found.
5. The number of neighbor minutiae about a centrally chosen focal minutia was found to be normally distributed with mean 6.42 and standard deviation 1.76.
6. The proportion of ending ridges found was in good agreement with Roxburgh [14], who used a comparable sampling region, but differed significantly from proportions found by Kingston [13], Sclove [16], Okajima [17], and Loesch [18]. These results demonstrate reproducibility in the proportion of ending ridges for the particular region studied, but also support the assertion that the proportion is, in general, highly variable.
7. Dot frequencies found were in agreement with Okajima [17], but differed from those found by Kingston [13] and by Sclove [16]. Differences in the regions, pattern types, and fingers sampled are a likely source of the discrepancies observed.
8. Minutia type frequencies were found to be independent of pattern type. This is in agreement with Okajima [17] and with Loesch [18].
9. Frequencies of minutia orientations on the right and left thumbs were found to exhibit a mirror image relationship, with an excess of positive orientations on the left hand and an equal proportion of excess negative orientations on the right hand.
10. Pooling of minutiae from both hands resulted in a combined frequency of 0.746 for the preferred minutia orientation. This value is in good agreement with Amy [8].

11. Minutia type and orientation were found to be virtually independent of one another.
12. The incidence of minutia types and orientations was found to be equal for corresponding positive and negative distances along the ridge flow.
13. Independence was observed among minutia type, orientation, and distance along the ridge flow.
14. Independence was not observed among ridge count, orientation, and type.
15. A linear relationship was found between minutia orientation and ridge count, with the proportion of the dominant minutia orientation increasing proximally.
16. Dot frequencies were not found to vary with ridge count.
17. The proportion of ending ridges among ending ridges and forks was found to vary significantly with ridge count, with the proportion of ending ridges decreasing proximally.
18. Ridge counts of one, zero, and negative one were found to show substantially greater proportions of forks than the other ridge counts. This observation is explained by a geometrically based restriction on the allowable combinations of minutia types and orientations, given these ridge counts.
19. After adjustment for ridge counts of one, zero, and negative one, a linear relationship was observed between the ridge count and the proportion of ending ridges, with the proportion of ending ridges decreasing proximally.
20. Modified linear models for minutia type and orientation frequencies allowed successful prediction of the observed frequencies on each ridge, but the low *p*-value obtained suggests that there may be other significant variables.

### Conclusion

The present work is an attempt to describe the distribution of epidermal ridge minutiae and the relationships among neighboring minutiae. For the first time some basic hypotheses regarding minutiae have been formulated and tested. Given the legal significance of epidermal ridges, it is surprising that this investigation should be a novel one. Of course, in the profession we have been successful at empirical application of decision rules in fingerprint comparison, but this success should neither discourage objective research nor lull us into dogmatic assertions of infallibility. It is hoped that this research will demonstrate the value of systematic study of minutiae to practitioners and that we may enter an era of objective research and honest intellectual inquiry into the partial fingerprint problem.

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